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TECHNICAL REPORT 400-75

Some General Results for the N-Port

D. J. R. Stock and L. J. Kaplan

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SOME GENERAL RESULTS FOR THE N-PORT

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ABSTRACT

A rectangular transfer matrix for the n-port ($n \geq 2$) network is developed here. Considered is a device so used that a p-port is transformed into a q-port ($p + q = n$ and $q \leq p$). Its rectangular transfer matrix has all the interconnection properties of usual transfer matrices, provided the matrices multiplied are conformable (this can also be expressed in circuit terms). Standard tests for reciprocity and losslessness are shown to be applicable to the generalized transfer matrix with some modification.

The single frequency decompositions of Kurss and Youla for the 2n-port are shown to be valid for the n-port.

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SOME GENERAL RESULTS FOR THE N-PORT

I. INTRODUCTION

The first occurrence of transfer matrices in electrical engineering utilized the chain, or ABCD, matrix. The introduction of the scattering matrix to circuit theory led to the definition of the transfer scattering matrix for the two-port. At this point of development the impedance, admittance, and scattering matrices of n-ports were well-known. However, the transfer matrices in the impedance and scattering formalism were restricted to the two-port. It has since been shown in the literature that if the device has any even number (2n) of ports (one shall refer to it as an even port device), the transfer matrix of the 2n-port is defined where the scalars in the two-port case are n x n matrices. Redheffer[1] has even considered the case where the elements were operators in Hilbert space. There has developed much literature on the 2n-port network. This formalism is especially pertinent at microwave frequencies for multimode propagation in a waveguide. Nevertheless, it is desirable to obtain a formulation that can consider both odd port devices and even port devices which are not used as 2n-ports (i.e., a four-port considered as transforming a three-port into a one-port).

The general single frequency lossy two-port (reciprocal) was shown by Wheeler[4] to be decomposable into a pure simple lossy section (two-port) bracketed by two lossless two ports (see fig. 1). Similarly it was shown by Haus[5] that the non-reciprocity of a two-port can be isolated in a similar manner (see fig. 1). For lossless networks Weissfloch[6] decomposed the two-port into an ideal transformer between two lossless transmission lines (see fig. 1).

For the single frequency 2n-port Kurss[7] has shown that the lossy 2n-port can be decomposed into the simple lossy 2n-port bracketed by two lossless 2n-ports. He also shows that the non-reciprocal network is amenable to similar treatment. Youla[8] showed that the lossless 2n-port can be decomposed into n uncoupled transformers between lossless all-pass networks, (i. e., if the scattering matrix is partitioned into

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

where each submatrix is n x n, S_{11} and S_{22} are the zero matrix. That is, there is no reflected wave, hence the terminology all-pass).

This paper considers the formulation at single frequency of a rectangular transfer scattering matrix to represent a network with p output and q input ports. The analysis is restricted to cases for which $p \geq q$.

II. FORMULATION OF THE TRANSFER SCATTERING MATRIX

To obtain this formulation, one begins by representing the device by an m x m scattering matrix: [S]. Although it is not a necessity, the scattering formalism shall be resorted to for the remainder of this paper. The device is considered to transfer a p-port into a q-port ($p + q = m$). Thus, partitioning [S]:

$$[S] = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} , \qquad (1)$$

where $[S_{11}]$ is q x q, $[S_{12}]$ is q x p, $[S_{21}]$ is p x q; $[S_{22}]$ is p x p. One also defines $\underline{b}_1$, $\underline{a}_1$ as q element column matrices and $\underline{b}_2$, $\underline{a}_2$ as p element column matrices. The device is represented by the following equations:

$$\underline{b}_1 = [S_{11}] \; \underline{a}_1 + [S_{12}] \; \underline{a}_2 , \qquad (2a)$$

$$\underline{b}_2 = [S_{21}] \; \underline{a}_1 + [S_{22}] \; \underline{a}_2 . \qquad (2b)$$

If the network were a 2n-port where p is equal to q(equals n), the inverse of S_{21} could be found (assuming its existence); otherwise the transfer matrix would not exist. For the case of interest here, however, $[S_{21}]$ is rectangular, requiring the following development.

One multiplies equation (2b) from the left side by the transpose of $[S_{21}]$: $[S_{21}]'$. $[S_{21}]'\ [S_{21}]$ is a square matrix; hence it may have an inverse[a] which is designated $[A]^{-1}$. Then one develops a 2q x 2p matrix [T] defined

---

a. It must be assumed that q is less than p in the non-square transfer matrix. If this were not true, [A] would not have an inverse since its rank must be, at most, the smaller rank of the two product matrices (as a consequence of the Binet-Cauchy formula[3]).* As a result of the preceding there is, at most, one non-square transfer matrix (i.e., there is no reverse transfer matrix as in the square case) for given values of p and q. There will always be a transformation between the output scattering matrix, [S], and the input scattering matrix, [W]:

$$[W] = [S_{11}] + [S_{12}]\ [S]\ \Big[[1] - [S_{22}]\ [S]\Big]^{-1} [S_{21}]$$

Alternatively,

$$W = \Big[[T_{11}]\ [S] + [T_{12}]\Big]\ \Big[[T_{21}]\ [S] + [T_{22}]\Big]'\ M^{-1}$$

where $M = [T_{21}S + T_{22}]\ [T_{21}S + T_{22}]'$.

The restriction that q is less than p may be stated more elegantly as a linear mapping not increasing the dimension of the space.

* F. R. Gantmacher, <u>The Theory of Matrices</u>, p. 12.

similarly to equation (2):

$$\begin{bmatrix} \underline{b}_1 \\ \underline{a}_1 \end{bmatrix} = [T] \begin{bmatrix} \underline{a}_2 \\ \underline{b}_2 \end{bmatrix} \tag{3}$$

The components of [T] are:

$$[T_{11}]_{q,p} = [S_{12}] - [S_{11}]\,[A]^{-1}\,[S_{21}]'\,[S_{22}] \quad ,$$

$$[T_{12}]_{q,p} = [S_{11}]\,[A]^{-1}\,[S_{21}]' \quad ,$$

$$[T_{21}]_{q,p} = -[A]^{-1}\,[S_{21}]'\,[S_{22}] \quad ,$$

$$[T_{22}]_{q,p} = [A]^{-1}\,[S_{21}]' \quad .$$

III. PROPERTIES OF TRANSFER MATRICES

The rectangular transfer matrix has most of the properties of the usual transfer matrices. If two transfer matrices are conformable, then matrix multiplication is possible. It should be pointed out here that the concept of cascading, implied by use of the T matrix, is not quite that of two-port practice, or of 2n-port practice, where half the ports, arbitrarily called "input," are to the left of a device, and half, called "output," are to the right. Here, since one deals with non-square matrices, the number of input ports differ from the number of output ports. Thus, a four-port with non-square T may connect to a one-port on its left, and a three-port on its right. However it is noted that a (3 + 2) - port can not be cascaded with a (2 + 4) - port using rectangular transfer matrices in this form.

In the 2n-port literature, the conditions for losslessness and reciprocity of the transfer matrix are formulated in terms of the 2n x 2n Pauli matrices:

$$[\sigma_1] = \begin{bmatrix} [0] & [1] \\ -[1] & [0] \end{bmatrix} ,$$

$$[\sigma_2] = \begin{bmatrix} [1] & [0] \\ [0] & -[1] \end{bmatrix} .$$

Where [0] and [1] are the n x n zero and n x n unit matrices, respectively.

For reciprocity,

$$[T]' \, [\sigma_1] \, [T] = [\sigma_1] ,$$

$$[T] \, [\sigma_1] \, [T]' = [\sigma_1] .$$

For losslessness,

$$[T]^* [\sigma_2] [T] = [\sigma_2] \quad ,$$
$$[T] [\sigma_2] [T]^* = [\sigma_2] \quad .$$

(* denotes complex conjugate transpose)

In the case of a network having a rectangular transfer scattering matrix [T], a necessary condition for reciprocity is:

$$[T] [\sigma_1] [T]' = [\sigma_1] \tag{3a}$$

while a necessary condition for losslessness is:

$$[T] [\sigma_2] [T]^* = [\sigma_2] \tag{3b}$$

It must be noted that these conditions are not sufficient. It is possible[a] to find scattering matrices which are not lossy or reciprocal that will map onto transfer matrices that satisfy conditions 3a and/or 3b. The mapping S → T is not even one-to-one for several S matrices may go into the same T (this is not true for square transfer matrices). Therefore later decompositions based on the T matrix will have an ambiguity which can only be resolved by resorting to the S matrix. (See section V on the Weissfloch equivalent circuit). However, in Appendix 2 it is seen that if [T] is reciprocal (lossless), then T T' (T T*) represents a reciprocal

a. See Appendix III.

(lossless) network. This also holds for what may be called pseudo-lossless forms; i.e., rectangular T matrices that satisfy 3b but represent lossy networks.

The third condition using σ_2:

$$T \, \sigma_3 \, T = \sigma_3 \quad , \quad \sigma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} ,$$

for symmetry does not carry over to the rectangular transfer matrix since the usual symmetry concept does not apply here.

IV. THE WHEELER TYPE DECOMPOSITIONS

The similarity of the isolation of the lossy part and the non-reciprocal part has been noted by Kurss, but this paper will only consider the isolation of the lossy segment. The general transfer matrix will be decomposed as follows:

$$[T] = [U_1]\,[C]\,[U_2] \quad . \tag{4}$$

U_1 and U_2 represent lossless networks while C is the simple lossy section; U_1 and C are m x n matrices, while T and U_2 are m x n matrices. Following Kurss (not in notation, however), one denotes the σ_2 adjoint of T, T^+:

$$[T]^+ = [\sigma_2]_m^{-1}\,[T]^*\,[\sigma_2]_n \quad .$$

One notes here that the σ_2 matrices are square and of different orders. This satisfies the usual relation:

$$(T_1 T_2)^+ = T_2^{\,+} T_1^{\,+} \quad .$$

A matrix is σ_2 self-adjoint if $T = T^+$, and a matrix represents a lossless network if it is σ_2 unitary.

It is noted that rectangular matrix products of the form AA' and A'A (with or without complex conjugates) have the same rank, and essentially the same eigenvalues and Jordan canonical form.[9] The difference is that the matrix of higher order has some additional zero eigenvalues to account for the difference in order.

Following Kurss one defines a matrix:

$$H = TT^{+} \quad . \tag{5}$$

Kurss defined the product in the reverse order, but the revised arrangement will keep H non-singular. When relation 4 is substituted into 5,

$$H = U_1 CC^{*} U_1^{-1} \quad .$$

It is seen that H has the same Jordan canonical form as CC^{*}. In addition,

$$U_2 = C^{-1} U_1^{-1} T$$

is a lossless network. Thus the constructive steps of Kurss required to obtain the constituants become:

1) Find the Jordan matrix of H.

2) Find a σ_2-unitary matrix U_1 such that

$$\theta_c = U_2 \theta \text{ is simple.}$$

θ is the matrix relating H and its Jordan matrix J ($H = \theta J \theta^{-1}$).

3) Choose C as a solution to the equation

$$CC^{*} = \theta_c J \theta_c^{-1}$$

The proof of Kurss for the feasibility of this constructive process carries through with slight modifications. Youla[2] has stated that the proof of Kurss, which assumes a diagonal Jordan matrix, can be extended so that this requirement is not needed. It would seem that this is applicable to the rectangular case also.

V. THE WEISSFLOCH DECOMPOSITION

The lossless network may be decomposed into the cascade of an all-pass 2q-port, "uncoupled transformers," and an all-pass 2p-port. It is noted that a lossless all-pass network can not be reciprocal for an odd-port device (i.e., all scattering coefficients on main diagonal cannot be zero).

To proceed with the proof one considers the following form: $TT^* = K$. This is a non-singular Hermitian matrix ($K^* = (TT^*)^* = (TT^*) = K$). K is also Hermitian positive semidefinite since it is of form TT^*, but since its determinant is non-zero, K is Hermitian positive definite. Then the proof proceeds in a manner similar to that of Youla[8] using the unique polar decomposition of a square matrix (T) into the product of a Hermitian positive matrix (G) and a unitary matrix (F), taken akin to complex number $Z = Ae^{i\Phi}$.[3a]

a. Page 277.

In this case the unitary matrix (F) is not required since K is Hermitian positive definite. Then following Youla's arguments exactly:

$$K = \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} \cosh\lambda & \sinh\lambda \\ \sinh\lambda & \cosh\lambda \end{bmatrix} \begin{bmatrix} U^* & 0 \\ 0 & V^* \end{bmatrix} ,$$

where U and V are q x q unitary matrices and cosh λ and sinh λ are diagonal real matrices.

If one defines a 2q x 2p matrix $\hat{T}$ so that $\hat{T}\,\hat{T}^* = \hat{T}\,\hat{T}' =$

$$\begin{bmatrix} \cosh\lambda & \sinh\lambda \\ \sinh\lambda & \cosh\lambda \end{bmatrix} ,$$

then T may be written:

$$T = \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \hat{T}\, W \quad .$$

Since T T* = K, W is unitary. The lossless condition applied to W requires that W be an all-pass. In the pseudo-lossless case W is not an all-pass and in fact, accounts for the loss in the overall network. Note that the matrix W is in no way specified by the matrix T (except for being unitary). It contains the information lost in the mapping S → T. In particular T does not determine whether the network is lossless or pseudo-lossless. Thus,

$$T = \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \hat{T} \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix} ,$$

where X and Y are p x p unitary matrices, and $\hat{T}$ is a 2q x 2p matrix representing ideal transformers, then

$$K = \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \hat{T}\,\hat{T}^{*} \begin{bmatrix} U^{*} & 0 \\ 0 & V^{*} \end{bmatrix} .$$

This shows that the Weissfloch decomposition is possible if one can construct a $\hat{T}$ such that,

$$\hat{T}\,\hat{T}' = \begin{bmatrix} \cosh\lambda & \sinh\lambda \\ \sinh\lambda & \cosh\lambda \end{bmatrix} .$$

A solution when p = n-1 and q = 1 is:

$$\hat{T} = \begin{bmatrix} \frac{1}{\sqrt{p}}\cosh\lambda \ldots \frac{1}{\sqrt{p}}\cosh\lambda & \frac{1}{\sqrt{p}}\sinh\lambda \ldots \frac{1}{\sqrt{p}}\sinh\lambda \\ \frac{1}{\sqrt{p}}\sinh\lambda \ldots \frac{1}{\sqrt{p}}\sinh\lambda & \frac{1}{\sqrt{p}}\cosh\lambda \ldots \frac{1}{\sqrt{p}}\cosh\lambda \end{bmatrix} .$$

If q is greater than one, there may be several possible matrices $\hat{T}$. This is shown by the following two acceptable matrices for q = 2 and p = 4.

$$\hat{T}_1 =$$

$$\begin{bmatrix} \cosh\lambda_1 & 0 & 0 & 0 & \sinh\lambda_1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & 0 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 \\ \sinh\lambda_1 & 0 & 0 & 0 & \cosh\lambda_1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & 0 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 \end{bmatrix}$$

$$\hat{T}_2 =$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}}\cosh\lambda_1 & \frac{1}{\sqrt{2}}\cosh\lambda_1 & 0 & 0 & \frac{1}{\sqrt{2}}\sinh\lambda_1 & \frac{1}{\sqrt{2}}\sinh\lambda_1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}\cosh\lambda_2 & \frac{1}{\sqrt{2}}\cosh\lambda_2 & 0 & 0 & \frac{1}{\sqrt{2}}\sinh\lambda_2 & \frac{1}{\sqrt{2}}\sinh\lambda_2 \\ \frac{1}{\sqrt{2}}\sinh\lambda_1 & \frac{1}{\sqrt{2}}\sinh\lambda_1 & 0 & 0 & \frac{1}{\sqrt{2}}\cosh\lambda_1 & \frac{1}{\sqrt{2}}\cosh\lambda_1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}\sinh\lambda_2 & \frac{1}{\sqrt{2}}\sinh\lambda_2 & 0 & 0 & \frac{1}{\sqrt{2}}\cosh\lambda_2 & \frac{1}{\sqrt{2}}\cosh\lambda_2 \end{bmatrix}$$

The ambiguity in $\hat{T}$ is complemented by complete non-specification of matrices X and Y. It is to be noted that reciprocity would require $V = \overline{U}$ and $Y = \overline{X}$.

APPENDIX I.

Figure 2 shows the transformer decompostion of the two six-ports whose matrices are given in the text. Figure 3 shows the complete decomposition of a lossless reciprocal three-port.

APPENDIX II: On The Forms $T' \sigma_1 T$ and $T^* \sigma_2 T$

For the 2n-port if T is reciprocal (lossless) T' and $\bar{T}$ are reciprocal (lossless). For the rectangular transfer matrix T' does not represent a network but the forms $T' \sigma_1 T$ and $T^* \sigma_2 T$ can be formed. Denoting: $A' S_{21}'$ by $S_{21}{}^{-1}{}_{\text{left}}$ it is seen that the form $T' \sigma_1 T(T^* \sigma_2 T)$ differs from $T \sigma_1 T'$ $(T \sigma_2 T^*)$ by the fact that the left inverse multiplies S_{21} on the right (hence in the 2n-port or square matrix case there is no difference). The results are subsumed in the following equations;

$$T' \sigma_1 T = \begin{bmatrix} 0 & S_{21} S_{21}{}^{-1}{}_{\text{left}} \\ -S_{21} S_{21}{}^{-1}{}_{\text{left}} & 0 \end{bmatrix} = \tau_1 \quad ,$$

$$T^* \sigma_2 T = \begin{bmatrix} (S_{21} S_{21}{}^{-1}{}_{\text{left}})^* S_{21} S_{21}{}^{-1}{}_{\text{left}} & 0 \\ 0 & -(S_{21} S_{21}{}^{-1}{}_{\text{left}})^* S_{21} S_{21}{}^{-1}{}_{\text{left}} \end{bmatrix} = \tau_2$$

The backward sigma matrices have the same rank and non-zero eigenvalues as the ordinary sigma matrices. They have additional zero eigenvalues to fill out their greater dimension.

A calculation will show if $T\ \sigma_1\ T' = \sigma_1$ then $T\ \tilde{\sigma}_1\ T' = \sigma_1$ and therefore $T\ T'$ would be a reciprocal 2n-port while $T'\ T$ would not. The similar result holds for the lossless case.

APPENDIX III

The n-port, with $n \geq 3$, that has this scattering matrix

$$S = \left[\begin{array}{cccc} 0 & 1 & 0 & \cdots \\ \hline 1 & 0 & 0 & \cdots \\ \cline{2-4} 0 & 0 & \chi & \cdots \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \end{array}\right]$$

will have the following rectangular scattering matrix

$$T = \left[\begin{array}{cc} 1\ 0 \cdots & 0 \cdots \\ \hline 0 \cdots & 1\ 0 \cdots \end{array}\right]$$

This transfer matrix will satisfy the lossless condition even though χ may be such that the original network is lossy active or indefinite. This transfer matrix also satisfies the reciprocity condition even though if $n \geq 4$, χ can be non-symmetric and hence the circuit be nonreciprocal. All these networks with different scattering matrices, that depend on χ, will map onto the given transfer matrix.

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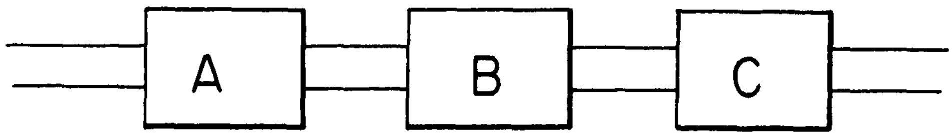


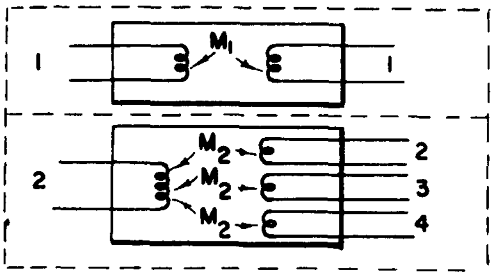
Fig. 1

The Decomposition of a Two-Port

In the Wheeler decomposition, B is pure lossy, while A and C are lossless. In Haus' decomposition, B is pure non-reciprocal, while A and C are reciprocal. In the Weissfloch transformer, B is an ideal transformer, while A and C are lengths of lossless transmission lines.

$$\hat{T}_1 =$$

$$\left[\begin{array}{cccccccc}
\cosh\lambda_1 & 0 & 0 & 0 & \sinh\lambda_1 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & 0 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda \\
\sinh\lambda_1 & 0 & 0 & 0 & \cosh\lambda_1 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & 0 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh
\end{array}\right.$$



$$\hat{T}_2 =$$

$$\left[\begin{array}{cccccccc}
\frac{1}{\sqrt{2}}\cosh\lambda_1 & \frac{1}{\sqrt{2}}\cosh\lambda_1 & 0 & 0 & \frac{1}{\sqrt{2}}\sinh\lambda_1 & \frac{1}{\sqrt{2}}\sinh\lambda_1 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}}\cosh\lambda_2 & \frac{1}{\sqrt{2}}\cosh\lambda_2 & 0 & 0 & \frac{1}{\sqrt{2}}\sinh\lambda_2 & \frac{1}{\sqrt{2}}\sinh\lambda \\
\frac{1}{\sqrt{2}}\sinh\lambda_1 & \frac{1}{\sqrt{2}}\sinh\lambda_1 & 0 & 0 & \frac{1}{\sqrt{2}}\cosh\lambda_1 & \frac{1}{\sqrt{2}}\cosh\lambda_1 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}}\sinh\lambda_2 & \frac{1}{\sqrt{2}}\sinh\lambda_2 & 0 & 0 & \frac{1}{\sqrt{2}}\cosh\lambda_2 & \frac{1}{\sqrt{2}}\cosh\lambda
\end{array}\right.$$

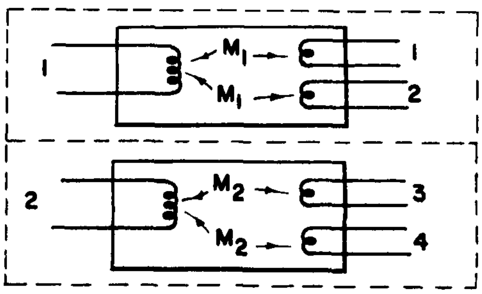


Fig. 2

The two possible couplings for a six-port operating as a four-to-two-port. Note that there is no coupling between the boxes. The matrices explicitly state the relations between the ports.

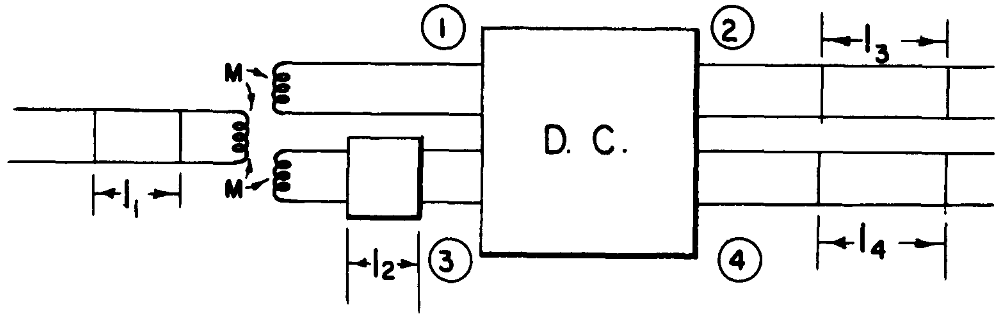


Fig. 3

The lossless three-port represented by a transformer (coupling M), four lengths of transmission line, and a directional coupler (D. C.) whose scattering matrix, using port numbersas shown above, is:

$$\begin{bmatrix} 0 & \alpha & 0 & j\beta \\ \alpha & 0 & j\beta & 0 \\ 0 & j\beta & 0 & \alpha \\ j\beta & 0 & \alpha & 0 \end{bmatrix}$$

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Unclassified Report

A rectangular transfer matrix for the n-port ($n \geq 2$) network is developed here. Considered is a device so used that a p-port is transformed into a q-port ($p+q=n$ and $q \leq p$). Its rectangular transfer matrix has all the interconnection properties of usual transfer matrices, provided the matrices multiplied are conformable (this can also be expressed in circuit terms). Standard tests for reciprocity and losslessness are shown to be applicable to the generalized transfer matrix with some modification.

The single frequency decompositions of Kurss and Youla for the 2n-port are shown to be valid for the n-port.

1. Non-Euclidean Geometry

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